Projection model equations

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| **Description** | **Variable or equation** | **Step** |
| Start populations | $$P\_{xgen}^{S1}$$ | (18.1) |
| Initial populations at risk | $$P\_{xgen}^{ARi}= P\_{xgen}^{Si}$$ | (18.2) |
| The fertility and nativity model for births | $$B\_{bgen}^{Si}= v\_{g}^{i}×\sum\_{x=10}^{x=49}f\_{xen}^{i}× P\_{xfen}^{ARi}$$ | (18.3) |
| Mixing: assigning ethnicity to the new-born | $$B\_{bgfn}^{Si}=\sum\_{e}^{}x\_{ef}^{i}×B\_{bgen}^{Si}$$ | (18.4) |
| Mortality model: when x <90 | $$D\_{xgen}^{i}= d\_{xgen}^{i}× P\_{xgen}^{ARi}$$ | (18.5) |
| Mortality model: when x >= 90 | $$D\_{xgen}^{i}=(1-s\_{xgen}^{i})× P\_{xgen}^{Si}$$ | (18.6) |
| Subtraction of special population stocks (prisoners and armed forces) | $$-C\_{xgen}^{i+}-A\_{xgen}^{i+}$$ | (18.7) |
| Emigration option (1) Exogenous projected emigration flows | $$E\_{xgen}^{i} $$ | (18.8) |
| Emigration option (2) Emigration (transmission) rates × Populations at Risk | $$E\_{xgen}^{i}= e\_{xgen}^{ti}× P\_{xgen}^{ARi}$$ | (18.9) |
| Emigration option (3) Emigration (admission) rates × Populations at Risk in the Rest of the World | $$E\_{xgen}^{i}= e\_{xgen}^{ai}× P\_{xgen}^{ARw(u)}$$ | (18.10) |
| Internal out-migration option (1) Multi-regional equation with constant or trended transition rates | $M\_{xgen}^{ij}(t)= m\_{xgen}^{ij}×$$P\_{xgen}^{ARi}$ (t) | (18.11) |
| Internal out-migration option (2) Adjustment of migration flow to destination shares of populations | $\left(\frac{{P\_{xgen}^{Sj}\left(t\right)}/{\sum\_{j}^{}P\_{xgen}^{Sj}\left(t\right)}}{{P\_{xgen}^{Sj}\left(ref\right)}/{\sum\_{j}^{}P\_{xgen}^{Sj}(ref)}}\right)× m\_{xgen}^{ij}\left(ref\right)×$ $P\_{xgen}^{ARi}$ | (18.12) |
| Internal out-migration option (3) Gravity model based on origin, destination and impedance factors | $$M\_{xgen}^{ij}\left(t\right)=a\_{0}+\sum\_{k}^{}a\_{k }X\_{k}^{i}+ \sum\_{l}^{}a\_{l} Y\_{i}^{j}+ f\left(c^{ij}\right)$$ | (18.13) |
| Total internal out-migrations are the sum of projected migration out-flows | $$M\_{xgen}^{i+}=\sum\_{j\ne i}^{}M\_{xgen}^{ij}$$ | (18.14) |
| Residual balances | $$R\_{xgen}^{i}= P\_{xgen}^{Si}-M\_{xgen}^{i+} - E\_{xgen}^{i}-C\_{xgen}^{i+}-A\_{xgen}^{i+}- D\_{xgen}^{i}$$ | (18.15) |
| Total internal in-migrations are the sum of projected migration in-flows | $$M\_{xgen}^{+i}=\sum\_{j\ne i}^{}M\_{xgen}^{ji}$$ | (18.16) |
| Immigration option (1)Externally generated projected immigration flows  | $$I\_{xgen}^{i}$$ | (18.17) |
| Immigration option (2)Immigration (transmission) rates × Population at Risk in Rest of the World | $$I\_{xgen}^{i}= i\_{xgen}^{ti}× P\_{xgen}^{ARw(u)}$$ | (18.18) |
| Immigration option (3)Immigration (admission) rates × Population at Risk  | $$I\_{xgen}^{i}= i\_{xgen}^{ai}× P\_{xgen}^{ARi}$$ | (18.19) |
| Addition of prisoners and armed forces | $$+C\_{xgen}^{+i}+A\_{xgen}^{+i}$$ | (18.20) |
| Final populations | $$P\_{xgen}^{Fi}= R\_{xgen}^{i}+ M\_{xgen}^{+i}+I\_{xgen}^{i}+C\_{xgen}^{+i}+A\_{xgen}^{+i}$$ | (18.21) |
| Populations at risk, convergence test | $$P\_{xgen}^{ARi}=0.5 × \left[P\_{xgen}^{Si}+ P\_{xgen}^{Fi}\right]$$ | (18.22) |
| Ethnic switching | $$P\_{xgfn}^{Fi}=\sum\_{e}^{}s\_{ef}^{i}×P\_{xgen}^{Fi}$$ | (18.23) |
| Ageing on | $$P\_{x+1gen}^{Si}= P\_{xgen}^{Fi}$$ | (24) |

Variable description

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| **Variable** | **Description** |
| *Stocks* | *Counts of people* |
| *PS* | Start Population in a time interval (count) |
| *PF* | Final Population in a time interval (count) |
| *A* | Armed Forces population |
| *C* | Prisoners |
| *Flows* | *Movements from one state to another* |
| *B* | Births  |
| *D* | Deaths  |
| *E* | Emigrations (international migration from UK to Rest of the World) |
| *M* | Migrations (internal to the country) |
| *Mij* | Migration from LAD i (origin) to LAD j (destination) |
| *Mi+* | Total out-migrations from LAD i = Σj≠iMij |
| *R* | Residual (balances) |
| *M+i* | Total in-migrations to LAD i = Σj≠iMji |
| *I* | Immigrations (international migration to the UK from the Rest of the World) |
| *Intensities* | *Either probabilities or occurrence-exposure rates*  |
| *f* | Fertility rates (occurrence exposure rates) for period-ages |
| *d* | Death rates (occurrence-exposure rates) for period-cohorts |
| *m* | Internal migration (transmission) rates |
| *eo* | Emigration (transmission) rates |
| *ea* | Emigration (admission) rates |
| *io* | Immigration (transmission) rates |
| *ia* | Immigration (admission) rates |
| *v* | Sex proportion at birth |
| *b* | Mixing probabilities of the ethnicity of a new-born given the ethnicity of mother |
| *s* | Switching probabilities of a new ethnicity a new ethnicity given ethnicity at a prior census |
| *Indexes* | *Subscripts or superscripts* |
| *x* | Age index (used for period-ages and period-cohorts) |
| *b* | Age index referring  |
| *g* | Gender (or sex) index  |
| *e* | Ethnic group  |
| *n* | Nativity group (birth place) |
| *i* | Zone index for zone of interest (origin) |
| *j* | Zone index for zone of interest (destination) |
| *z* | Zone index for the last zone in the system |
| *u(i)* | Zone index for rest of the UK |
| *w(u)* | Zone index for rest of world or rest of world region |
| *o* | Transmission rate = migration/origin population |
| *a* | Admission rate = migration/destination population |
| *t* | Stocks: a point in time; Flows: an interval in time from t to t+1 |
| *O* | Stocks: indicates removal (out) |
| *I* | Stocks: indicates addition (in) |