**Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS): ESRC-funded (2008-2012): RES-179-25-0009**

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**Study-Level Documentation**

The Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) project investigated ways of raising students’ attainment and engagement by using formative assessment to inform teaching and learning of mathematics in secondary school. The project focused on topics that are central to the current mathematics curriculum and to further progression and participation in mathematics, science and other STEM projects: algebra, ratio, decimals and fractions.

In Phase 1, we conducted a nationally representative survey of around 7000 students examining the understandings and attitudes of current students across Key Stage 3. This survey uses three tests first developed by the Concepts in Secondary Mathematics and Science study enabling a comparison with the understandings of students in the 1970s as well as a detailed analysis of current students’ mathematical understandings.

**Research Design and Methods**

*Test design*

The survey used tests of Algebra, Ratio and Decimals developed in the Concepts in Secondary Mathematics and Science (CSMS) study (Hart, 1981; Hart & Johnson, 1983). The Ratio test was supplemented by a small number of items focusing on multiplicative reasoning from the CSMS Fractions test. The CSMS tests were designed and trialled on the basis of results from diagnostic interviews, and the final versions were administered in 1976 and/or 1977. The focus was on conceptual understanding and application, although for completeness a very small number of items were designed to assess mathematical procedures. The tests used in the survey are available on the project website:

*Coding and data analysis*

The coding system is described in the original CSMS project publications and technical reports (Hart, 1981; Hart, et al, 1985, Hart & Johnson, 1983) In the 1970s analysis, items were selected empirically from each test to form a series of hierarchical levels of difficulty. The number of levels identified varies from test to test. Hierarchies in the Algebra test were identified using 30 out of the 51 items to construct 4 levels, in the Ratio test using 20 out of the 27 items to again form 4 levels, and in the Decimals test using 39 out of 72 items to form 6 levels.

Students were judged to have been successful at a specific level if they had successfully answered two-thirds or more of the items at that level. Students who had not achieved two-thirds of Level 1 items were said to be at ‘Level 0’. It was possible to broadly describe the type of mathematical understanding required for the items in each level in each topic, although these were not always neat descriptions since the items and levels were assigned mainly on empirical rather than theoretical grounds.

*Participants*

Over two summers in 2008 and 2009, tests were administered to a sample of approximately 7000 students across Key Stage 3 (KS3) from 19 schools randomly selected from the MidYIS database (Tymms & Coe, 2003; http://www.cemcentre.org/midyis/introduction).

Each student took two of the three tests so as to provide comparative information between tests but without overloading students. The numbers of students in each year-group taking each test is therefore roughly two-thirds of the total number of students involved in that age group.

Hart, K. (Ed.). (1981). *Children's Understanding of Mathematics: 11-16.* London: John Murray.

Hart, K., Brown, M., Kerslake, D., Küchemann, D. E., & Ruddock, G. (1985). *Chelsea Diagnostic Mathematics Tests: Teacher's Guide*. Windsor: NFER-Nelson.

Hart, K. M., & Johnson, D. C. (Eds.). (1983). *Secondary school children's understanding of mathematics: A report of the mathematics component of the concepts in secondary mathematics and science programme*. London: Centre for Science Education, Chelsea College.

Tymms, P. & Coe, R. (2003) Celebration of the success of distributed research with schools: the CEM Centre, Durham. *British Educational Research Journal*, 29 (5), 639-653

*Research ethics*

The research was approved by the King’s College London Research Ethics Committee (REC) for Education and Management (Protocol Number: REP(EM)/07/08-9). The datasets are anonymised. Students, completing tests and questionnaires, were informed verbally about the research by their teachers. They were informed that participation was voluntary and that they had the right to withdraw from the research at any time (until December 2009) and without giving a reason. They were given an information sheet about the research. Students’ parents were informed about the project in accordance with each schools’ normal policy, normally either by letter or through a school’s newsletter to parents.

**Data-level documentation**

Three SPSS .sav files are provided with test results for the three tests [algebra.sav, ratio.sav, numbers.sav] together with SPSS syntax used to create the data files:

$alg format cross sectional data for ESRC archive.sps [Algebra]

$num format cross sectional data for ESRC archive.sps [Numbers]

$rat format cross sectional data for ESRC archive.sps [Ratio]

Detailed data level documentation is provided within these files.

Samples of the tests are attached.

A description of variables applicable to all three datasets together with details of the items and correct responses follows:

|  |  |  |
| --- | --- | --- |
| Variable Name | Label  | Additional Information |
| test\_ID | Unique test response identifier | Anonymised |
| test\_type | Type of test |  |
| test\_year | Year in which test taken |  |
| test\_date | Date test taken (imputed from set/batch if missing) |  |
| sch | School code | Anonymised |
| phase | Phase in ICCAMS project |  |
| year | School year group (grade) when test taken |  |
| PUPIL\_ID | ICCAMS unique pupil identifier | This can be used to match pupils different test scores |
| sex | Sex (1=m, 2=f) |  |
| dob | Date of birth (date format) | Month and year only to anonymise the data. |
| MidYIS\_score | Overall MidYIS score | National sample [N(103, 15)] |
| weight | Weight applied to give nationally representative sample | A weighting exercise has been carried out to make each year group follow the national distribution [N(103, 15)] using 20 bins. |
| num\_total | Total score on Numbers items |  |
| L1 | CSMS Level 1 achieved |  |
| L2 | CSMS Level 2 achieved |  |
| L3 | CSMS Level 3 achieved |  |
| L4 | CSMS Level 4 achieved |  |
| L5 | CSMS Level 5 achieved |  |
| L6 | CSMS Level 6 achieved |  |
| HiLev | CSMS level |  |

**Algebra: Variable Names, Item descriptors and Correct Responses**

|  |  |  |  |
| --- | --- | --- | --- |
| Variable Name | Label – Item Descriptor | Hierarchy Item / Level | Correct Response |
| alg01aa\_r | If x? x + 2, 6 ->? |  | 8, +8 |
| alg01bb\_r | If x? 4x, 3 ->? |  |  |
| alg01cc\_r | If x? x + 2, c ->? |  |  |
| alg02ab\_r | Smallest and largest of n+1, n+4, n-3, n, n-7 |  |  |
| alg03ab\_r | Which is larger, 2n or n + 2 ? Explain why | 4 |  |
| alg04aa\_r | Add 4 to 8 |  |  |
| alg04bb\_r | Add 4 to n+5 |  |  |
| alg04cc\_r | Add 4 to 3n | 3 |  |
| alg04dd\_r | Mult 8 by 4 |  |  |
| alg04ee\_r | Mult n+5 by 4 | 4 |  |
| alg04ff\_r | Mult 3n by 4 |  |  |
| alg05aa\_r | If a+b=43, a+b+2=? | 1 |  |
| alg05bb\_r | If n-246=761, n-247=? |  |  |
| alg05cc\_r | If e+f=8, e+f+g=? | 3 |  |
| alg06aa\_r | a, if a+5=8 | 1 |  |
| alg06bb\_r | b, if b+2=2b |  |  |
| alg07aa\_r | Area of rect 3 x 4 |  |  |
| alg07bb\_r | Area of rect 6 x 10 | 1 |  |
| alg07cc\_r | Area of rect n x m | 2 |  |
| alg07dd\_r | Area of rect 5 x (e+2) | 4 |  |
| alg08@@\_r | Perimeter, if sides 9+1+10+2 | 1 |  |
| alg09aa\_r | Perimeter, if sides e+e+e | 1 |  |
| alg09bb\_r | Perimeter, if sides h+h+h+h+t | 2 |  |
| alg09cc\_r | Perimeter, if sides u+u+5+5+6 | 2 |  |
| alg09dd\_r | Perimeter, if n sides of length 2 | 3 |  |
| alg10aa\_r | c=#carrots@8p, t=#turnips@6p, 8c+6t stands for? |  |  |
| alg10bb\_r | c=#carrots@8p, t=#turnips@6p, total no of veg=? |  |  |
| alg11aa\_r | u=v+3, v=1, u=? | 2 |  |
| alg11bb\_r | m=3n+1, n=4, m=? | 2 |  |
| alg12@@\_r | John has J marbles, Petra has P marbles, tot no of marbles=? |  |  |
| alg13aa\_r | Simplify 2a+5a | 1 |  |
| alg13bb\_r | Simplify 2a+5b | 3 |  |
| alg13cc\_r | Simplify (a+b)+a |  |  |
| alg13dd\_r | Simplify 2a+5b+a | 2 |  |
| alg13ee\_r | Simplify (a-b)+b | 4 |  |
| alg13ff\_r | Simplify 3a-(b+a) |  |  |
| alg13gg\_r | Simplify a+4+a-4 |  |  |
| alg13hh\_r | Simplify 3a-b-a | 3 |  |
| alg13ii\_r | Simplify (a+b)+(a-b) |  |  |
| alg14@@\_r | If r=s+t,r+s+t=30, r=? | 3 |  |
| alg15aa\_r | Diagonals = sides-3, 57 sides ? ? | 2 |  |
| alg15bb\_r | Diagonals = sides-3, k sides ? ? | 3 |  |
| alg16@@\_r | If c+d=10, c<d, what can you say about c? | 3 |  |
| alg17aa\_r | £200/day + £20/hr overtime, h=hrs overtime, P=total pay, write equation connecting P and h | 4 |  |
| alg17bb\_r | £200/day + £20/hr overtime, h=hrs overtime, P=total pay, if h=4, P=? |  |  |
| alg18aa\_r | A+B+C=C+A+B: true always, never, sometimes? |  |  |
| alg18bb\_r | L+M+N=L+P+N: true always, never, sometimes (when?)? | 4 |  |
| alg19aa\_r | a=b+3, b inc by 2, what happens to a? |  |  |
| alg19bb\_r | f=3g+1, g inc by 2, what happens to f? |  |  |
| alg20@@\_r | Cakes cost c pence each and buns cost b pence each. If I buy 4 cakes and 3 buns, what does 4c + 3b stand for? | 4 |  |
| alg21@@\_r | If (x + 1)^3 + x = 349 when x=6, solve (5x + 1)^3 + 5x = 349 | 4 |  |
| alg22@@\_r | Total cost of b blue pencils @ 5p & r red pencils @ 6p is 90p. Write equation in b & r | 4 |  |
| alg23ab\_r | Machine does +10 then x5. Equivalent to x?? then +?? |  |  |
| alg24@@\_r | a, b, and d are positive whole numbers. When is a/b < a/d? |  |  |
| alg25aa\_r | What does \* mean in spreadsheet formula? |  |  |
| alg25bc\_r | Which is larger, "=2+(100\*A5)" or "=100+(2\*A5)"? Explain why |  |  |
| alg26aa\_r | Value of "=B3+8" if B3=20 |  |  |
| alg26bb\_r | Name of spreadsheet cell |  |  |
| alg26cc\_r | Formula to give 5 times value in D4 |  |  |

**Ratio: Variable Names, Item descriptors and Correct Responses**

|  |  |  |  |
| --- | --- | --- | --- |
| Variable Name | Label – Item Descriptor | Hierarchy Item / Level | Correct Response |
| rat01a1\_r | 521: The 2 stands for 2  |  |  |
| rat01a2\_r | 521 400: The 2 stands for 2  |  |  |
| rat01b1\_r | 0.2: The 2 stands for 2 ... |  |  |
| rat01b2\_r | 0.260: The 2 stands for 2  |  |  |
| rat01b3\_r | 0.412: The 2 stands for 2 ... |  |  |
| rat02a1\_r | Write in figures: Two hundred and fifty |  |  |
| rat02a2\_r | Write in figures: Four hundred thousand and seventy three |  |  |
| rat02a3\_r | Write in words: 8030 |  |  |
| rat02a4\_r | Write in words: 140 000 |  |  |
| rat02b1\_r | Write in words: 0.29 |  |  |
| rat02b2\_r | Add ten to 3597 |  |  |
| rat02b3\_r | Add ten to 0.15 |  |  |
| rat02b4\_r | Add one hundred to 21 534 |  |  |
| rat03ac\_r | Add one hundred to 19 930 |  |  |
| rat04ad\_r | Add one tenth to 4.254 |  |  |
| rat04av\_r | Add one tenth to 2.9 |  |  |
| rat04ah\_r | Take away one hundred from 583 |  |  |
| rat04bb\_r | Take away one hundred from 30 000 |  |  |
| rat05@@\_r | Identify the number on a ruler 7 tenths of the way from 20 to 30 |  |  |
| rat06ab\_r | Identify the number on a ruler 8 tenths of the way from 5 to 6 |  |  |
| rat06cd\_r | Identify the number on a ruler 4 tenths of the way from 2.7 to 2.8 |  |  |
| rat07aa\_r | Identify the number on a ruler 6.5 tenths of the way from 14 to 15 |  |  |
| rat07bb\_r | Identify the number on a ruler 1 fifth of the way from 3 to 4 |  |  |
| rat08aa\_r | Identify the number on a ruler about a third of the way from 7 to 8 |  |  |
| rat08bb\_r | Counter at 06399, after one more will be? |  |  |
| rat08cc\_r | 2 less than 17 000? |  |  |
| rat08dd\_r | Is there any difference between 4.9 and 4.90? Why? |  |  |
| fra02@@\_r | Which is bigger, 0.75 or 0.8? Why? |  |  |
| fra09aa\_r | Which is bigger, 20 100 or 20 095? |  |  |
| fra09bb\_r | Which is bigger, 7.55 or 7.5? |  |  |
| fra09cc\_r | Which is bigger, 4.06 or 4.5? |  |  |
| fra09dd\_r | Write as a decimal three hundredths |  |  |
| fra09ee\_r | Write as a decimal eleven thousandths |  |  |
| fra15@@\_r | Write as a decimal eleven tenths |  |  |
| fra16aa\_r | Four tenths is the same as ?? hundredths |  |  |
| fra16bb\_r | Write down any number between 4000 and 5000 |  |  |
| fra18@@\_r | Write down any number between 4100 and 4200 |  |  |
| fra19@@\_r | Write down any number between 0.4 and 0.5 |  |  |
| fra21@@\_r | Write down any number between 0.41 and 0.42 |  |  |
| fra22@@\_r | How many different numbers lie between 0.41 and 0.42? |  |  |
| fra24@@\_r | Shaded area of 2 units and 3 tenths |  |  |

**Number: Variable Names, Item descriptors and Correct Responses**

|  |  |  |  |
| --- | --- | --- | --- |
| Variable Name | Label – Item Descriptor | Hierarchy Item / Level | Correct Response |
| num01aa\_r | 521: The 2 stands for 2  |  |  |
| num01bb\_r | 521 400: The 2 stands for 2  |  |  |
| num02aa\_r | 0.2: The 2 stands for 2 ... |  |  |
| num02bb\_r | 0.260: The 2 stands for 2  |  |  |
| num02cc\_r | 0.412: The 2 stands for 2 ... |  |  |
| num03aa\_r | Write in figures: Two hundred and fifty |  |  |
| num03bb\_r | Write in figures: Four hundred thousand and seventy three |  |  |
| num04aa\_r | Write in words: 8030 |  |  |
| num04bb\_r | Write in words: 140 000 |  |  |
| num04cc\_r | Write in words: 0.29 |  |  |
| num05aa\_r | Add ten to 3597 |  |  |
| num05bb\_r | Add ten to 0.15 |  |  |
| num05cc\_r | Add one hundred to 21 534 |  |  |
| num05dd\_r | Add one hundred to 19 930 |  |  |
| num05ee\_r | Add one tenth to 4.254 |  |  |
| num05ff\_r | Add one tenth to 2.9 |  |  |
| num05gg\_r | Take away one hundred from 583 |  |  |
| num05hh\_r | Take away one hundred from 30 000 |  |  |
| num06aa\_r | Identify the number on a ruler 7 tenths of the way from 20 to 30 |  |  |
| num06bb\_r | Identify the number on a ruler 8 tenths of the way from 5 to 6 |  |  |
| num06cc\_r | Identify the number on a ruler 4 tenths of the way from 2.7 to 2.8 |  |  |
| num06dd\_r | Identify the number on a ruler 6.5 tenths of the way from 14 to 15 |  |  |
| num06ee\_r | Identify the number on a ruler 1 fifth of the way from 3 to 4 |  |  |
| num06ff\_r | Identify the number on a ruler about a third of the way from 7 to 8 |  |  |
| num07@@\_r | Counter at 06399, after one more will be? |  |  |
| num08@@\_r | 2 less than 17 000? |  |  |
| num09ab\_r | Is there any difference between 4.9 and 4.90? Why? |  |  |
| num10ab\_r | Which is bigger, 0.75 or 0.8? Why? |  |  |
| num10cc\_r | Which is bigger, 20 100 or 20 095? |  |  |
| num10dd\_r | Which is bigger, 7.55 or 7.5? |  |  |
| num10ee\_r | Which is bigger, 4.06 or 4.5? |  |  |
| num11aa\_r | Write as a decimal three hundredths |  |  |
| num11bb\_r | Write as a decimal eleven thousandths |  |  |
| num11cc\_r | Write as a decimal eleven tenths |  |  |
| num11dd\_r | Four tenths is the same as ?? hundredths |  |  |
| num12aa\_r | Write down any number between 4000 and 5000 |  |  |
| num12bb\_r | Write down any number between 4100 and 4200 |  |  |
| num12cc\_r | Write down any number between 0.4 and 0.5 |  |  |
| num12dd\_r | Write down any number between 0.41 and 0.42 |  |  |
| num12ee\_r | How many different numbers lie between 0.41 and 0.42? |  |  |
| num13aa\_r | Shaded area of 2 units and 3 tenths |  |  |
| num13bb\_r | Shaded area of 1 unit, 4 tenths and 3 hundredths |  |  |
| num13cc\_r | Shaded area of 1 unit and 7 hundredths |  |  |
| num14aa\_r | Multiply 4 by ten |  |  |
| num14bb\_r | Multiply 5.13 by ten |  |  |
| num14cc\_r | Multiply 317 by one hundred |  |  |
| num14dd\_r | Multiply 2.3 by one hundred |  |  |
| num14ee\_r | Divide 1600 by one hundred |  |  |
| num14ff\_r | Divide 3.7 by one hundred |  |  |
| num14gg\_r | Divide 24 by twenty |  |  |
| num14hh\_r | Divide 16 by twenty |  |  |
| num15ac\_r | Which is bigger, 8x4 or 8/4? 8x0.4 or 8/0.4? 0.8x0.4 or 0.8/0.4? |  |  |
| num16aa\_r | Column addition: 263 + 978 |  |  |
| num16bb\_r | 13.4 + 2.7 = |  |  |
| num16cc\_r | Column subtraction 2312 - 547 |  |  |
| num16dd\_r | 8.44  6.37 = |  |  |
| num17aa\_r | 20 × 500 = |  |  |
| num17bb\_r | 5 × 0.2 = |  |  |
| num17cc\_r | 0.2 × 0.4 = |  |  |
| num17dd\_r | 60 ÷ 3 = |  |  |
| num17ee\_r | 60 ÷ 0.3 = |  |  |
| num18aa\_r | Which is nearest in size to 182? 100 or 82 or 180 or 150 or 200 or 190 |  |  |
| num18bb\_r | Which is nearest in size to 2.9? 3 or 30 or 2 or 20 or 0 or 1 |  |  |
| num18cc\_r | Which is nearest in size to 0.18? 0.1 or 10 or 0.2 or 20 or 0 or 1 or 2 |  |  |
| num18dd\_r | Which is nearest in size to 2.9x7? .002 or .02 or .2 or 2 or 20 or 200 or 2000 |  |  |
| num18ee\_r | Which is nearest in size to 0.29x7.1? .002 or .02 or .2 or 2 or 20 or 200 or 2000 |  |  |
| num18ff\_r | Which is nearest in size to 59÷190? .003 or .03 or .3 or 3 or 30 or 300 or 3000 |  |  |
| num19aa\_r | What is the calculation: A table is 92.3 centimetres long. About how many inches is this? (1 inch is about 2.54 centimetres.) |  |  |
| num19bb\_r | What is the calculation: My car tank was full after I put in 6.44 gallons of petrol. The tank holds 8.37 gallons. How much petrol was in it to start with? |  |  |
| num19cc\_r | What is the calculation: The price of pebbles is shown as 88.2 pence for each kilogram. What is the cost of a bag containing 0.58 kg of pebbles? |  |  |
| num19dd\_r | What is the calculation: The cost of 6.22 litres of petrol was £4.86. What would the price of one litre be? |  |  |
| num19ee\_r | What is the calculation: My car can go 41.8 miles on each gallon of petrol on a motorway. How many miles can I expect to travel on 8.37 gallons? |  |  |
| num20@@\_r | Write a story which matches this sum: 6.4 + 2.3 = 8.7 |  |  |